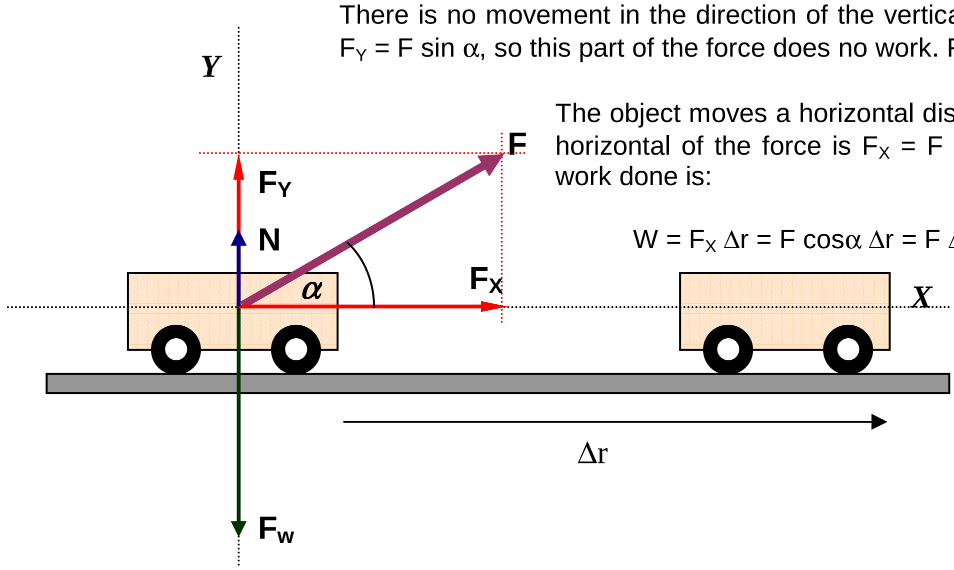
WORK, ENERGY, POWER
Revision Guide

Work ***W*** is done when a *force* ***F*** moves through a *distance* ***Δr***.

There is no movement in the direction of the vertical component, $F_Y = F \sin \alpha$, so this part of the force does no work. $F_Y + N = F_w$

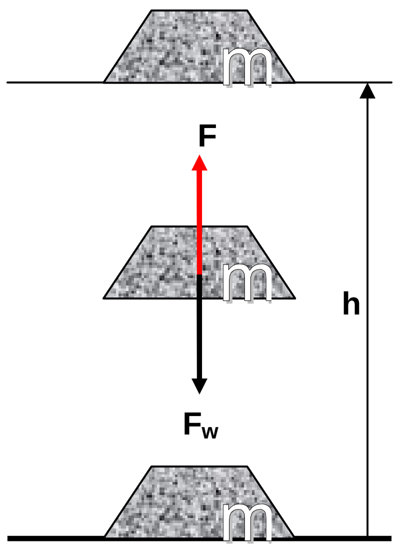
The object moves a horizontal distance Δr. The horizontal of the force is $F_X = F \cos \alpha$, so the work done is:

$$W = F_X \Delta r = F \cos\alpha \, \Delta r = F \, \Delta r \cos\alpha$$



$$W = F \, \Delta r \cos \alpha$$

Gravitational potential energy ***E_{Pg}*** is the energy that an object have because of its position. An object possesses **gravitational potential energy** if it is above a possible equilibrium position. How much gravitational potential energy does a given object have? It depends on how much work would have to be done to lift it above an equilibrium position:



Work must be done to lift the mass against "the pull of gravity." An upward force moves the mass upwards.

How much work is done?

$$W = F \, \Delta r \cos\alpha$$

$F = F_w = mg$ (the weight of the object)
$\Delta r = h$ (the height of elevation)
$\alpha = 0^o \rightarrow \cos\alpha = 1$

$$W = mgh$$

How much potential energy does the object now have? Exactly as much work as was done to elevate it!

$$E_{Pg} = mgh$$

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Elastic potential energy ***E*$_{Pe}$** is energy stored as a result of deformation of an elastic object, such as the *stretching* Δl of a spring with an elastic constant k.

$$E_{Pe} = \frac{1}{2} k\, \Delta l^2$$

Kinetic energy ***Ek*** is the energy that an object possesses due to its motion. To analyze the relationship between work and kinetic energy we shall consider the uniformly accelerated motion that results when a constant *resultant force* ***F*** acts on some *mass* m that is initially at rest. If the mass m accelerates to a *final velocity* ***v*** in some *displacement* ***Δr*** as a result of the force, the work done by the resultant force on the mass:

$$W = F\, \Delta r \cos\alpha$$

According to Newton's second law of motion: $\sum F = m\, a$. And $v^2 = v_0^2 + 2\, a\, \Delta r$. Therefore, the work done by the resultant force:

$$W = F\, \Delta r \cos\alpha = m\, a\, \Delta r \cos 0 = m \left(\frac{v^2}{2\, \Delta r} \right) \Delta r = \frac{1}{2} m\, v^2$$

For a *mass* ***m*** travelling at a *speed* ***v*** the kinetic energy:

$$E_K = \frac{1}{2} m\, v^2$$

The units of work and energy are the joule (J)

Energy is conserved. That is, energy cannot be created or destroyed, but it can be converted from one form into another **(principle of the conservation of energy).**

Mechanical energy is *conserved* when the forces are conservative. All work done by a conservative force is stored as kinetic energy E_K and potential energy E_P in a system and is completely recoverable. Then:

$$E_K + E_P = K \longrightarrow E_{K,1} + E_{P,1} = E_{K,2} + E_{P,2}$$

Mechanical energy is *lost* when a non-conservative (dissipative) force does work. When non-conservative forces are present the conservation of mechanical energy equation must be modified to:

$$E_{K,0} + E_{P,0} = E_{K,F} + E_{P,F} + W_{nc}$$

where W_{nc} is the work done by the non-conservative forces.

Power ***P*** is the time rate of doing work or the time rate of using energy. If an amount of work ***W*** is done (or energy E used) in an elapsed time ***t***, the average power:

$$P = \frac{W}{t}$$

The SI unit for power is the watt (w) and the engineering unit is the *horsepower* (HP).